No Smooth Shortcut: Why Field Equations Cannot Represent Irreducible Computation

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Abstract

Continuous field equations—the mathematical backbone of General Relativity, Maxwell electrodynamics, and Quantum Field Theory—operate within mathematical structures that are "tame" in the model-theoretic sense: they cannot define the integers, successor functions, or infinite discrete structure. But von Neumann machines are physical systems that demonstrably perform computations requiring exactly these structures. This creates a forced incompatibility: either field equations are incomplete for physical reality, or the laptop on your desk is not really computing. We formalize this as a minimal axiomatic proof using o-minimality theory and the von Neumann machine as empirical anchor, producing a trilemma: accept incompleteness, deny physical computation, or abandon tameness in favor of discrete mathematical structure. Crucially, a companion proof [1] establishes that the "deny computation" escape route leads to its own impossibility: a timeless block ontology that cannot accommodate the generative structure of consequential truth. Together, the two proofs form a decision tree with no cost-free exits—accepting computational irreducibility forces discrete structure; denying it eliminates becoming, computation, and contingency simultaneously.

Central Puzzle. A silicon chip executes AES-256 encryption. This is a physical process—electrons moving through gates under electromagnetic field governance. Maxwell's equations describe the substrate completely, or so the completeness claim goes. But AES-256 requires 14 rounds of irreducible substitution, permutation, and mixing over a discrete state space of 2^{256} possibilities, with a definable successor relation between rounds. The mathematical structure of field equations cannot define this successor relation or this discrete state space. So where, in the field-equation description of reality, does the computation live?

Core Thesis. Continuous field equations, operating within tame (o-minimal) mathematical frameworks, lack the definability to represent the discrete computational structures that von Neumann machines physically instantiate. This is not a philosophical claim about emergence or interpretation—it is a mathematical impossibility result about what these equations can express.

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Formal Strategy. Two axioms, five lemmas, one theorem, forced choice. Accept the conclusion or accept an absurdity.

Reasoning Model—Forced Incompatibility. The argument proceeds by showing that three intuitively appealing commitments cannot be jointly maintained:

(A) Physical computation is ontologically real (von Neumann machines actually compute).

(B) Continuous field equations completely describe the physical domain containing these machines.

(C) The mathematical framework of field equations is tame (o-minimal or comparably restrictive).

The theorem establishes: $\mathbf{A} \wedge \mathbf{B} \wedge \mathbf{C} \rightarrow \perp$. Therefore at least one of **A**, **B**, or **C** must be rejected. Since rejecting **A** is computational eliminativism and rejecting **C** concedes discrete structure in physics, the most natural exit is rejecting **B**—accepting that field equations are incomplete for computational phenomena.

Key Structural Insight. A complete physical theory must represent the generative structure of computation *internally*—within its own mathematical language—not merely describe computational outcomes *externally* through an observer's interpretive overlay. A theory whose ontology contains only smooth fields but whose "computational content" exists solely in an external agent's interpretation of those fields has not described the computation; it has described the substrate and left the computation to someone else. This is the deep reason why simulation is insufficient (Lemma 4) and emergence cannot rescue completeness (Lemma 5).

Independence from Temporal Metaphysics. This proof does not depend on block time arguments, truthmaker theory, or the metaphysics of becoming. The constraint is purely mathematical: what can o-minimal structures define? The answer excludes what von Neumann machines require. This holds whether one is an eternalist, presentist, or agnostic about time. For the complementary temporal argument, see [1].

1 Key Definitions

Definition (Von Neumann Machine). *A physical device implementing the von Neumann architecture—sequential instruction fetch, decode, execute over discrete memory states. Every modern general-purpose computer is one. These machines are physical systems governed by physics.*

Definition (Irreducible Computation). *A computational process whose outcome cannot be obtained without performing work equivalent to executing the process—no shortcut exists that bypasses the stepwise state transitions while preserving the input-output mapping. AES encryption, SHA-256 hashing, and constraint-satisfaction solving are canonical examples.*

Definition (O-Minimal Structure). *A mathematical structure expanding the ordered real field* $(\mathbb{R}, +, \times, <)$ *in which every definable subset of* $\mathbb{R}$ *is a finite union of points and open intervals. This is the formal characterization of "tame" mathematics. Standard* *result [2, 3]: no o-minimal expansion of $\mathbb{R}$ can define $\mathbb{Z}$, successor functions on infinite sets, or any infinite discrete subset with definable structure.*

Definition (Field Equations)**.** *Partial differential equations over smooth manifolds governing physical fields: Einstein field equations (GR), Maxwell's equations (electrodynamics), Yang-Mills equations (gauge theories), Dirac/Klein-Gordon equations (QFT). All operate over continuous domains using smooth or distributional tensor fields.*

Definition (Tameness Condition)**.** *The conditional assumption that a physical theory's mathematical framework maintains o-minimal definability constraints—i.e., it does not define infinite discrete structure. This is a* conditional *premise, not an assertion about what physics "really" does. The proof shows what follows if this condition holds. This result applies to theories whose complete physical ontology is representable within a tame continuous mathematical structure; it does not rule out theories that explicitly incorporate non-tame or discrete generative primitives—indeed, the proof's conclusion is that such primitives may be necessary.*

2 Axioms

Axiom 1 (Physical Computation Exists)**.** *Von Neumann machines are physical systems. They perform computation. The computation is a physical process—not merely a human interpretation projected onto electron movements, but a real structural feature of what the system does. Denying this requires accepting that no physical system actually computes anything.*

Rejection Cost: If computation is not physically real—if it is purely interpretive overlay—then "computer" is an arbitrary label we attach to certain electron configurations with no more physical significance than seeing faces in clouds. Your laptop does not compute; it merely moves electrons, and "computing" is a story you tell about it. This is the eliminativist position, and it is the price of denying Axiom 1.

Axiom 2 (Completeness Requires Representational Adequacy)**.** *If a physical theory claims to completely describe a domain of physical reality, its mathematical framework must be able to represent all physically real structures and relationships within that domain. A theory that cannot represent what a physical system does within its domain is incomplete for that system.*

Rejection Cost: Denying this means accepting that a physical theory can be "complete" while being unable to describe physical processes occurring within its domain. This empties "completeness" of meaning.

3 Lemmas

Lemma 1 (Von Neumann Machines Require Discrete Computational Structure)**.** *The operation of a von Neumann machine constitutively requires:*

(i) Discrete state space—*memory states drawn from a finite but combinatorially vast discrete set.*

(ii) Successor relation—*each computational step produces a unique next state determined by the current state and instruction.*

(iii) Discrete branching—*conditional logic (if/then/else) creating discrete forks in execution path.*

(iv) Threshold transitions—*gate logic producing discontinuous output (0 or 1) from continuous input signals.*

These are not incidental features or approximations. They are what makes the system a computer rather than a resistor or a heat sink.

Proof. By the definition of von Neumann architecture: instruction fetch requires a program counter with successor (PC $\rightarrow$ PC $+ 1$), memory access requires discrete addressing (address $\in \{0, 1, \ldots, 2^n - 1\}$), and ALU operations require discrete logic gates with threshold behavior. These are constitutive—a system lacking them does not implement the architecture. $\square$

Lemma 2 (O-Minimal Structures Cannot Define Discrete Computational Structure)**.** *No o-minimal expansion of the ordered real field* $(\mathbb{R}, +, \times, <)$ *can define:*

(i) The integers $\mathbb{Z}$ *as a subset of* $\mathbb{R}$*.*

(ii) A successor function $S: X \to X$ *on any infinite discrete set* X*.*

(iii) Any infinite discrete subset of $\mathbb{R}$ *with definable internal structure.*

(iv) Any function exhibiting infinitely many discontinuous threshold transitions.

Proof. Standard model theory. By the o-minimality definition, every definable subset of $\mathbb{R}$ is a finite union of points and intervals. $\mathbb{Z}$ is not such a union. The graph of a successor function on an infinite discrete set is an infinite discrete subset of $\mathbb{R}^2$, which cannot be definable in any o-minimal structure (definable subsets of $\mathbb{R}^n$ in o-minimal structures have finite cell decompositions). See van den Dries [2], Ch. 1–3; Pillay and Steinhorn [3]. $\square$

Lemma 3 (Field Equations Operate in Tame Mathematical Structures)**.** *The standard mathematical frameworks of continuous field theories—smooth manifolds, tensor fields, differential operators, Sobolev spaces, distributional solutions—satisfy the tameness condition: they do not define infinite discrete structure within their natural mathematical setting.*

Conditional Framing: This lemma is presented conditionally. If the mathematical framework of a field theory maintains tameness (does not introduce definability of $\mathbb{Z}$ or successor), then Lemma 2 applies to that framework.

Why Tameness Is a Natural Constraint for Physics: Tameness is not an arbitrary restriction imposed to manufacture a result. It captures four properties that physicists independently require of well-behaved theories: (i) *empirical measurability*—physical quantities are extracted from continuous measurements with finite precision, not from detecting whether a value is an integer; (ii) *stability under perturbation*—physical predictions should not depend on infinitely precise discrete conditions that vanish under any perturbation; (iii) *finite information access*—no finite experiment can determine membership in $\mathbb{Z}$ versus a dense subset of $\mathbb{R}$; (iv) *exclusion of definitional encoding*—physically meaningful structure should not depend on tricks that smuggle arithmetic into geometry. Theories that violate tameness permit pathological definable sets that undermine the geometric and analytic methods field theories depend on.

Support: The structures used in GR (smooth Lorentzian manifolds), classical electrodynamics (smooth vector bundles), and standard QFT (operator-valued distributions on smooth manifolds) are all naturally modeled in o-minimal or tame expansions of $\mathbb{R}$. Even distributional solutions and mild singularities remain within definability constraints—they do not introduce successor functions or infinite discrete subsets as definable objects.

Strategic Note: A physicist who rejects this lemma faces a dilemma, not an escape. If they claim field equations operate in a mathematical framework that *can* define $\mathbb{Z}$ and successor functions—i.e., one that is not tame—they are conceding Option C before the proof even reaches the theorem: physics requires discrete mathematical structure beyond standard continuous field theory. If they accept the framework is tame, Lemma 2 applies. Either way, the proof forces a choice. □

Lemma 4 (Simulation Is Not Definable Representation)**.** *A continuous dynamical system can* simulate *a Turing machine—via thresholding, bistable attractors, symbolic dynamics, and similar techniques. But simulation is not the same as definable representation within the theory's mathematical framework.*

The distinction:

- **Simulation**: The continuous system's trajectory, when *externally interpreted* by an observer who imposes a discrete coding scheme, corresponds to steps of a computation. The discrete structure exists in the interpretation, not in the equations.
- **Definable representation**: The discrete computational structure (successor relation, state space, branching) is expressible within the mathematical language of the theory itself, without external interpretive scaffolding.

Clarification—Definability vs. Interpretability: A model theorist might object that "interpretability" is weaker than "definability"—perhaps the theory can *interpret* a discrete structure without *defining* it as a subset. This distinction is real but does not rescue the objection. What the proof requires is not syntactic interpretability (a coding trick that maps terms to terms) but *truthmaker-adequate representation*: the theory's ontology must contain the actual infinite discrete chain of states with preserved order and causal/entailment structure. A syntactic reinterpretation that codes discrete states into continuous parameters without preserving the successor *as a real structural relation in the theory's ontology* does not meet Axiom 2's adequacy requirement—it provides a code, not a description of what the system is doing.

Proof. Consider a continuous dynamical system $\dot{x} = f(x)$ whose trajectory, under an externally imposed partition of state space, simulates a Turing machine. The partition that discretizes the continuous trajectory into computational steps is not definable within the o-minimal structure of the ODE—it requires the external imposition of a discrete coding. The continuous equations describe a smooth flow; the "computation" exists only relative to an interpretation that the equations themselves cannot express. This is the difference between a vinyl record (continuous groove) and the music (structured information requiring a decoding scheme the groove cannot define). □

Lemma 5 (Emergence Does Not Rescue Completeness)**.** *If discrete computational structure is said to "emerge" from continuous field dynamics, then exactly one of the following holds:*

(a) *The emergent computation is fully reducible to the continuous description—every discrete computational fact is equivalent to some continuous field configuration, and the continuous theory completely captures it. Then computational irreducibility is illusory: there exists a continuous shortcut that bypasses the discrete steps while preserving the outcome. AES-256 encryption becomes, in principle, solvable by a continuous field equation without performing 14 rounds. Cryptographic security is not a physical fact but a limitation of our discrete framing.*

(b) *The emergent computation is not fully reducible—the discrete computational structure is a genuine feature of physical reality that the continuous description cannot completely capture. Then the continuous theory is incomplete.*

Proof. These exhaust the logical space. Either the continuous theory captures the computational structure (a) or it does not (b). Under (a), the computational irreducibility that makes AES-256 secure is an artifact of discrete description, not a physical fact. Under (b), the continuous theory fails the adequacy requirement of Axiom 2. No intermediate position is coherent: one cannot maintain that computation is both physically real and irreducible while also maintaining that the continuous theory completely describes it, because "completely describes" means the computational facts are derivable from the continuous description, which means they are reducible to it. □

4 Theorem and Proof

Theorem 1 (Field Equations Cannot Completely Describe Physical Computation)**.** *Under the tameness condition, continuous field equations cannot provide a complete description of physical systems performing irreducible computation.*

Proof. (1) Von Neumann machines are physical systems that perform irreducible computation (Axiom 1).

(2) Their operation constitutively requires discrete computational structure: successor relations, discrete state spaces, threshold transitions, discrete branching (Lemma 1).

(3) A complete physical theory must represent all physically real structures within its domain (Axiom 2).

(4) Therefore, any field theory claiming complete description of electromagnetic systems (which include von Neumann machines) must be able to represent discrete computational structure within its mathematical framework (from 1, 2, 3).

(5) Under tameness, the mathematical framework of continuous field equations cannot define discrete computational structure—no successor relations, no $\mathbb{Z}$, no infinite discrete sets, no threshold functions (Lemmas 2, 3).

(6) Invoking simulation does not help: simulation requires external interpretive scaffolding not definable within the theory (Lemma 4).

(7) Invoking emergence does not help: either computation reduces to continuous description (destroying irreducibility) or it does not (conceding incompleteness) (Lemma 5).

(8) From (4) and (5): the field equations must represent what they mathematically cannot represent.

(9) Contradiction. Therefore, under tameness, continuous field equations cannot completely describe physical systems performing irreducible computation. □

5 The Forced Choice

The theorem forces exactly one of three positions. There is no fourth option.

Option A: Accept Incompleteness. Continuous field equations are incomplete for physical systems that compute. They correctly describe electromagnetic substrates but cannot represent the computational processes those substrates physically perform. Field equations stand to computation as classical mechanics stands to quantum phenomena: empirically successful within scope, foundationally incomplete beyond it.

Cost: Modest. Requires acknowledging that physics needs mathematical tools beyond smooth manifolds and PDEs to describe everything physical systems do. Discrete and hybrid frameworks (causal set theory, loop quantum gravity, cellular automata) become not just theoretical curiosities but necessary complements to continuous field theory.

Option B: Deny Physical Computation. Computation is not physically real. The laptop on your desk does not actually compute—it merely moves electrons through silicon in patterns that humans *interpret* as computation. No physical system computes anything; computation is always and only interpretive overlay.

Cost: Catastrophic. This is *computationally eliminativist.* It entails that cryptographic security has no physical basis (it is interpretive), that the Church-Turing thesis describes nothing physical, that computer science is not a science of physical systems, and that the observable differences between a working computer and a broken one are not computational differences at any level of physical description.

Further Cost—No Safe Harbor: Denying ontological computational irreducibility commits one to a timeless block ontology in which computation and physical evolution are non-generative—all facts exist timelessly and completely, independent of any process of generation. This is precisely the position analyzed in the companion proof [1], which demonstrates that such a position must additionally deny that becoming is ontologically real, that computation is ontologically productive, and that contingency is genuine. Option B does not escape the impossibility; it routes the defender into a second impossibility that has already been mapped. The two proofs form a fork: accept irreducibility and discrete generative structure is forced (this proof); deny irreducibility and becoming, computation, and contingency evaporate [1]. There is no branch that preserves continuous completeness, ontological irreducibility, *and* timeless block time simultaneously.

Option C: Abandon Tameness. Continuous field equations must be extended with mathematical structures that violate o-minimality—introducing definable integers, successor functions, or infinite discrete structure into the fundamental mathematical framework of physics.

Cost: Significant but honest. This means the smooth-manifold, PDE-based framework of modern physics is not the whole story at the level of mathematical foundations. It means discrete structure is not emergent but fundamental, and continuous field equations are approximations of something deeper that includes discrete elements.

No Fourth Option. Every escape attempt collapses into one of these three:

- "Computation is emergent" $\rightarrow$ Either reduces to (A) or (B), per Lemma 5.
- "Field equations can represent computation through simulation" $\rightarrow$ Equivocates on "represent"; see Lemma 4. This is (B) in disguise.
- "O-minimality doesn't apply to physics" $\rightarrow$ This *is* Option C.
- "Computational irreducibility is epistemic only" $\rightarrow$ This *is* Option B.
- "I deny ontological irreducibility entirely" $\rightarrow$ This *is* Option B with additional downstream commitments (see [1]).

6 Objections and Blocks

Objection 1: "Continuous systems can simulate Turing machines." *Response*: Correct and irrelevant. Simulation requires an external observer to impose a discrete coding scheme on continuous trajectories. The discrete structure is in the interpretation, not in the equations. See Lemma 4. If simulation suffices, one has adopted Option B: the computation exists only in interpretation, not in physical reality.

Objection 2: "Tameness is too strong / physics isn't literally o-minimal." *Response*: The proof is explicitly conditional. Rejecting tameness for physics is choosing Option C—conceding that physics requires mathematical structure beyond tame continuous frameworks. This is a *concession*, not a rebuttal.

Objection 3: "Computation is just a high-level description of physical processes." *Response*: This is Option B. One is saying computation is not physically real—it is descriptive overlay.

Objection 4: "Distributional solutions, singularities, and non-smooth structures handle discreteness." *Response*: Distributional solutions (Schwartz distributions, Sobolev spaces) extend smooth function spaces but remain within tame mathematical frameworks—they do not introduce definable successor functions or definable infinite discrete subsets. A Dirac delta is not a successor function. If these features *do* provide definability of $\mathbb{Z}$ and successor, one is choosing Option C.

Objection 5: "Quantum mechanics already introduces discreteness." *Response*: This is an important partial concession that deserves careful treatment. Standard QFT formulates quantum mechanics on continuous Hilbert spaces and smooth spacetime manifolds. Quantum measurement produces discrete outcomes, but the measurement process itself raises the key question: *does it break tameness?*

Consider three prominent interpretations. Under *decoherence* (without objective collapse), the fundamental dynamics remain unitary and continuous—discrete outcomes are perspectival, arising from entanglement with the environment. The underlying mathematical structure stays tame; the discreteness is in the observer's branch, not the equations. This does not rescue completeness, because the discrete computational structure of the von Neumann machine must still be represented somewhere, and the continuous Schrödinger evolution cannot define it (Lemma 2 applies to Hilbert space operators on smooth manifolds just as to classical fields).

Under *objective collapse* theories (GRW, Penrose), the wavefunction undergoes genuine stochastic discontinuities. If these collapses introduce definable discrete structure into the theory's mathematical framework—definable successor relations, discrete state spaces with internal structure—then collapse theories have *abandoned tameness*. This is Option C: physics requires non-tame mathematical structure. The proof succeeds by forcing the concession.

Under *many-worlds*, all branches are equally real and the dynamics are fully continuous. The discrete outcomes in a given branch are indexical, not ontological. The completeness problem for computation remains: where, in the universal wavefunction's continuous evolution, is the successor relation that makes AES-256 require 14 rounds?

In all cases, if quantum discreteness provides the necessary computational structure, one is either (a) acknowledging that the continuous field equation framework is insufficient on its own (Option A/C), or (b) identifying a specific mechanism by which quantum processes break tameness—still a concession that continuous field equations alone are incomplete.

Objection 6: "This conflates levels of description." *Response*: The proof does not conflate levels. It operates on a single conditional chain: if field equations claim complete description of electromagnetic systems, and von Neumann machines are electromagnetic systems that compute, then the field equations must be able to represent the computation. The "levels" response implicitly concedes that field equations do not describe computation—placing it at a "different level" unreachable from the field equations. This is Option A (incompleteness) with extra vocabulary.

Objection 7: "Lattice regularizations, effective field theories, and hybrid models already handle discreteness." *Response*: This objection conflates computational technique with ontological commitment. Lattice QCD, for example, discretizes spacetime as a *regularization*—a computational tool for extracting predictions from a continuum theory. The lattice is scaffolding; the theory's ontological commitments remain continuous (the continuum limit is the physical theory, the lattice is an approximation to it). If lattice regularization were taken as ontologically fundamental—if the discrete lattice *is* the physics, not an approximation to something continuous—this would be a version of Option C: physics requires discrete structure at the foundational level.

Effective field theories (EFTs) similarly operate within the tame continuous framework at each energy scale. They introduce cutoffs, but cutoffs are parameters in continuous theories, not definable discrete structure in the o-minimal sense. An EFT does not define $\mathbb{Z}$ or successor functions within its mathematical language; it parameterizes ignorance about higher-energy continuous dynamics.

Hybrid models that explicitly incorporate both continuous and discrete elements—such as quantum gravity proposals with discrete Planck-scale structure coupled to continuous low-energy fields—are precisely the kind of theory the proof's conclusion motivates. They are not counterexamples; they are instances of Option A or Option C, acknowledging that purely continuous mathematics is insufficient. The proof's force is in showing this acknowledgment is *necessary*, not optional.

7 Relationship to Block Time Argument

This proof is logically independent of the block time impossibility proof [1] but complementary in force.

	Block Time Proof [1]	Field Equations Proof
Target	Completed block time (all truths co-exist timelessly)	Continuous field equations (completeness claims)
Anchor	Sudoku constraint-resolution	Von Neumann machine executing AES-256
Mathematical tool	Truthmaker theory, constitutive dependence	O-minimality, definability constraints
Nature of impossibility	Temporal: generated truths require generative ordering	Representational: tame structures cannot define discrete computation
Applies to	Any domain claiming timeless co-existence of generated truths	Any continuous field theory under tameness

Combined Force—The Decision Tree. The two proofs form a complete decision tree with no cost-free exits:

- Does ontological computational irreducibility exist?
- **Yes** → This proof applies: field equations under tameness cannot represent it. Accept incompleteness (Option A) or abandon tameness (Option C).
- **No** → The block time proof [1] applies: all facts exist timelessly; becoming is not ontological; computation is non-generative; contingency is illusory.

Either way, the defender of continuous field equation completeness must make an explicit, costly commitment. No position preserves all of: continuous completeness, ontological irreducibility, and timeless block time. The two proofs close each other's escape routes.

8 Implications

For Physics. This is not an attack on the empirical success of field equations. Maxwell's equations are extraordinarily successful. The claim is narrower: they are incomplete. They describe the electromagnetic substrate of computation without describing the computation itself. This is analogous to classical mechanics being empirically successful while being incomplete for quantum phenomena—not wrong, but not the whole story.

Discrete and hybrid frameworks—causal set theory [7], loop quantum gravity [5], cellular automata models—naturally avoid this impossibility because their mathematical foundations include discrete structure. This result provides theoretical motivation for these approaches beyond their usual quantum-gravity context: discrete structure may be needed not just for Planck-scale physics but for the adequate physical description of computation at any scale.

For Dark Matter, Dark Energy, and Quantum Foundations. If continuous field equations are representationally incomplete for computational phenomena, this raises a question: might other anomalies in physics—dark matter, dark energy, quantum measurement—also reflect the limits of continuous field theory rather than unknown continuous fields or particles? This proof does not establish this, but it removes the assumption that continuous field equations *must* be adequate for all physical phenomena and opens theoretical space for alternatives.

For Computer Science. If Option B is rejected (as it should be), computation is physically real and requires adequate physical description. This grounds computational complexity theory in physical reality—P $\neq$ NP is not merely a mathematical conjecture but a claim about what physical systems can and cannot do. The Church-Turing thesis becomes a physical thesis, not merely a mathematical one.

Boundary: What This Proof Does Not Refute. This argument does not refute timeless Platonism or mathematical eternalism as general metaphysical positions. What this proof demonstrates is that such views, if combined with continuous field theory as a complete physical description, must reject ontological computational irreducibility outright. The proof does not say Platonism is wrong; it says Platonism-plus-continuous-completeness has a specific, visible cost that must be paid explicitly rather than evaded.

What This Proof Establishes—Precise Statement. If physical reality contains processes whose outcomes require irreducible computational work—work that cannot be bypassed without destroying the input-output relationship—then any complete physical theory of those processes must possess mathematical structure capable of representing discrete successor relations, discrete state spaces, and threshold transitions. Tame continuous field-equation frameworks, by established model-theoretic results, cannot supply such structure. Therefore, such frameworks cannot be complete descriptions of physical reality if ontological computational irreducibility exists. This is a conditional, mathematical, and defensible result.

A Technical Notes on O-Minimality

Definition [2]: A structure $\mathcal{M} = (\mathbb{R}, <, +, \times, \ldots)$ expanding the ordered real field is *o-minimal* if every definable subset of $\mathbb{R}$ (in any number of parameters) is a finite union of points and open intervals.

Key results used:

1. *No definable* $\mathbb{Z}$: In any o-minimal structure, $\mathbb{Z}$ is not a definable subset of $\mathbb{R}$ (immediate from definition).

2. *No definable successor on infinite sets*: Any definable function in an o-minimal structure is piecewise continuous; a successor function on an infinite discrete set is nowhere continuous, hence not definable.

3. *Cell decomposition*: Every definable set in an o-minimal structure admits a finite cell decomposition. Infinite discrete subsets of $\mathbb{R}^n$ with definable structure cannot have finite cell decompositions.

4. *Standard examples*: $(\mathbb{R}, <, +, \times)$ is o-minimal (Tarski). $(\mathbb{R}, <, +, \times, \exp)$ is o-minimal [4]. These include the mathematical settings of standard field theories.

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